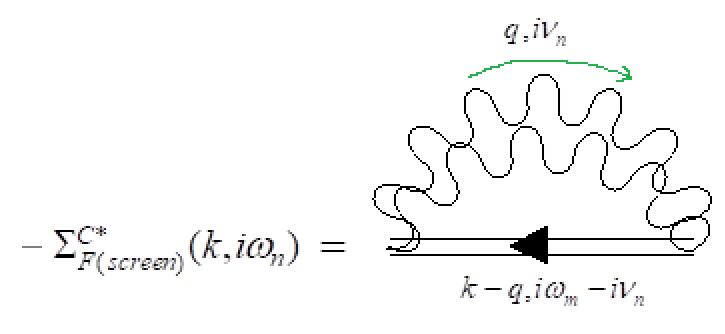
**Single Particle Excitations**

Now let’s do a few calculations. We’ll start with the main contribution to our self energy now,



**Scattering time near kF within the screened (RPA) Fock approximation, ΣF(screen)**

Luttinger’s theorem is that the scattering time for a **k**-state electron approaches nill as **k** approaches **k**F, at least for the e-e interaction (and that coincidentally GC(k,ω) - real time causal GF - has a discontinuity at kF too, just like it does in the non-interacting case). We can calculate it for the case of a Fock screened interaction self-energy term (this is a little more general than our Σ at the top of the page as we’re replacing G0 with G, but I’m going to undo this in a second).



(remember I’m presuming momentum going against arrows) So,



(where GC\* is the the GF with ΣF(screen)C\* as its self-energy. In order to obtain an estimate of the quasi-particle lifetime, let us replace GC\* with G0C\*, and then we’ll just have our simple above estimate accurate to O(V2):



Let’s try to evaluate this quantity. I’m going to be using some properties of εRPA that we’ll get into later I guess. Since |εRPA(q,z)| ~ |z|2 for large z, we can evaluate S using the contour integral method. Note εRPA(q,z) will have zeros on the real axis (‘cause there are plasma oscillations there up until the point where it acquires and imaginary part’). To avoid finding the zeroes of εRPA(q,z) on the real axis, we introduce a branch cut. Then, where f(z) is the S summand (see Stat Mech folder, Math Appendix):



where we have the last line, since



Remember our GF, with the self-energy, is of form, GC\*(k,iωn) = 1/[iωn – ξk – ΣC\*(k,iωn)]. To get the lifetime we need ImΣR(k,ω) and to first order, we can approximate the self-energy by simply evaluating it at ω = ξk. So then analytically continuing iωn → ω + iη, and making the approximation, we have:



So then we have:



Now take the T → 0 limit,



Then



which means only a small shell of q-space contributes to the lifetime. Also, for small |ω|.



So,



Using the Lindhard form,



We see that at T = 0,



which gets contributions only from a shell of k-space inside the Fermi surface. Combining everything,



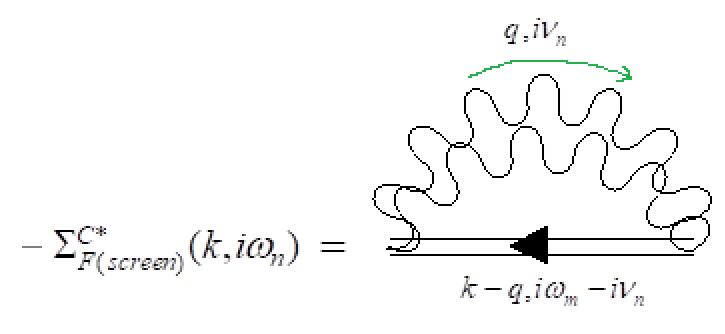
In 3D, the integrals give,



Remember that ξk measures energies from the chemical potential (or Fermi surface at T = 0). The key point is that as k approaches the Fermi surface, the lifetime proceeds to ∞. This holds up even for more accurate calculations. This property validates the quasiparticle concept and underlies the notion of a Landau Fermi Liquid, that we can treat even a strongly interacting gas’s (basically a liquid then) excitations as being particle like, near the Fermi surface.

**Energy corrections within the screened (TF) Fock approximation, ΣF(screen)**

Now let’s look at the real part of the self-energy to see how the new and improved self-energy affects the single particle spectrum. So go back to:



and,



Again going to replace GC\* with G0C\*, and then we’ll just have our simple above estimate accurate to O(V2), which is our expression at the top of the page :



Let’s try to evaluate this quantity. But to make it simpler, I’m going to use the TF approximation to ε, instead of the RPA approximation,



where we’ll recall from the Free Day/Electrons/Equilibrium Properties file that:



Well, evaluating S(q,iωn),



Filling this into Σ, we get:



And so this calculation is the same one we made previously w/in the Hartree Fock approximation, just using the screened potential energy V(q)/εTF(q) instead of the bare Coulomb potential energy V(q). So we’ll parallel those calculations. First, I’ll write our result (in 3D) as,



and then note:



So,



Now we’ll try for the radial integration.



So we need two anti-derivatives,



and,



So….



That is horrible. The colored guys cancel.



and,



and putting everything together,



which is,



And we can factor out a kF,



Let’s define:



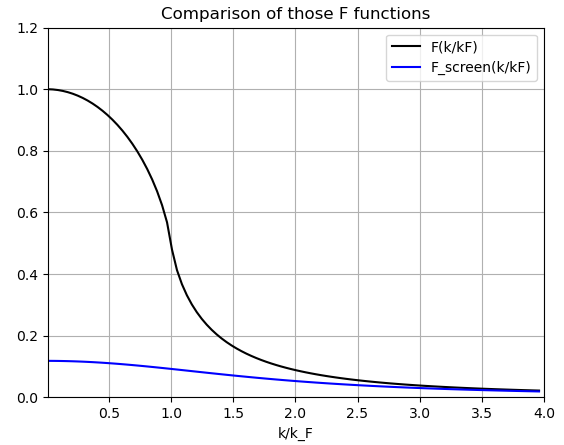
Then we can say,



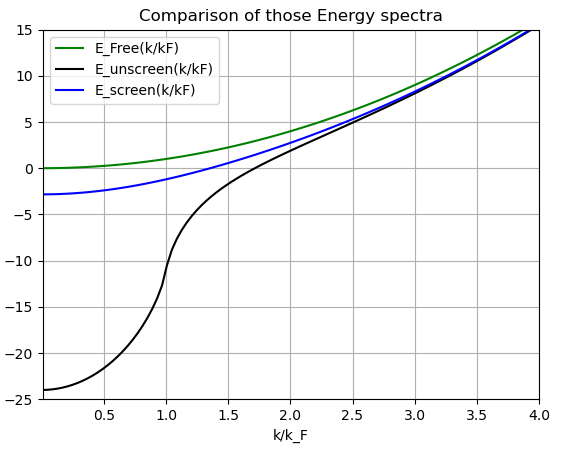
which, reassuringly, reduces to the unscreened Σ when qTF = 0. And up to this order, our approximation for the renormalized single electron energy levels would be:



Let’s see what this looks like, for TF = 1.5,



And a comparison of the energies: the free particle spectrum, unscreened spectrum from previous file, and screened spectrum,



So ultimately, the self-energy arising from the e-e interaction gives a modest correction for most metals. We’ll see this conclusion reinforced when we study Fermi Liquid Theory in the context of 3He, which could also apply here. One more thing. So we’ve been approximating the energy spectrum by going to the GF,



and looking for the pole,



by expanding Σ about ω = ξk (which is where we’d expect it to be, to zeroth order)



It’d be a little more accurate to go further out in the expansion. We could say,



where the ´ denotes differentiation w/r to ω. Taking note that ξk = k2/2m – μ, we could interpret the denominator as, among other things, renormalizing the mass, to:



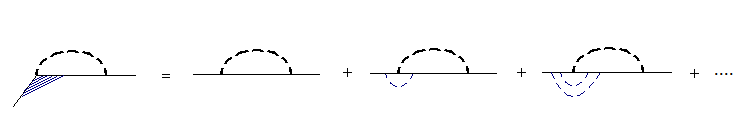
I guess this would be like a ‘running mass renormalization’ since it’d depends on k. I think people usually just take the value at k = kF (which makes ξk = 0). So we’d say,



Since I used the Thomas-Fermi approximation to ε, there was not derivative to take. So to evalute this, we’d have to use the Lindhardt approximation at least. Turns out when we do, the mass correction is < 5% I think.

**Vertex Corrections (Hubbard Approximation)**

The ΣF(screen) term takes better account of the actual interaction between pairs of electrons. I think it’s equivalent to calculating the PE of electrons distributed according to the free many-body wavefunction, interacting via a screened interaction. Additional improvements to our self-energy estimate, to the Fock term in particular, usually take the form of vertex corrections. I’m not sure if these improve the model of the interaction, or of the distribution of the electrons – or particularly of the correlations between the spin up/down electrons. These take the form of the following set of diagrams:



(bold dotted line is screened interactions) Recall from the Hubbard approximation that the vertex corrections basically take a multiplicative role. So we just say,



I’m calling it FSH for Fock screened Hubbard? Where we’ll recall,



So the vertex corrections to the self energy are just the Gamma function there that we already analyzed. We don’t put a vertex on both ends because that is over counting. However if the scattering described weren’t from indistinguishable electrons like we have here, then it would be legitimate to put in two Gamma functions.

**e-e interaction vs. phonon, impurity interactions**

The e-e interaction renormalizes the electron energy spectrum, basically by changing its effective mass m → m\*. The effective mass can be measured from low temperature specific heat measurements. The mass corrections are usually negligible compared to those from other sources:

* band structure (usually fairly small in metals, larger in semiconductors)
* electron-phonon interactions (typically 10% - 150%) enhancement in metals
* scattering from local magnetic moments (often up to 1000%) for ‘heavy – fermion’ systems.

**Random Comment about Dimensional Effects**

As we lower the dimensions of the sample, the e-e interaction plays a larger and larger role. PT converges less well in 2D, and in 1D completely breaks down – successive terms in the expansion are larger than first order terms. In lower dimensions, it appears that the e-e interactions completely wipe out any single particle concept for any density, and any interaction strength. nk­ becomes flat for instance.

But anyway, insofar as 3D is concerned…the e-e interaction’s primary effect is to simply renormalize the free-electron parameters.